

## BRIEF REPORT

## Unconscious Addition: When We Unconsciously Initiate and Follow Arithmetic Rules

François Ric  
University of BordeauxDominique Muller  
Pierre Mendes France University at Grenoble and University  
Institute of France

This research shows that people can unconsciously initiate and follow arithmetic rules (e.g., addition). Participants were asked to detect whether a symbol was a digit. This symbol was preceded by 2 digits and a subliminal instruction: *add* or a control instruction. Participants were faster at identifying a symbol as a number when the symbol was equal to the sum of the 2 digits and they received the instruction to add the digits, suggesting that people can unconsciously solve arithmetic problems. Experiments 2 and 3 replicate these findings and demonstrate that the underlying processes can operate when the to-be-added digits are not perceived consciously. Thus, the unconscious can do (at least simple) arithmetic, such as addition.

*Keywords:* goal activation, action initiation, subliminal priming, unconscious processes, arithmetic

The Western philosophical tradition has taught that most important actions and decisions are governed by conscious deliberative thinking. However, this point of view has been challenged as studies have begun to show that activities that were long assumed to be the exclusive product of consciousness can also be run unconsciously (see Bargh, 2007; Dijksterhuis & Aarts, 2010). Dijksterhuis and colleague (2004; Dijksterhuis & Nordgren, 2006) went one step further with their unconscious thought theory and showed that for complex decisions, unconscious thinking can sometimes be more efficient than conscious thinking. Yet due to the recentness of this exploration, not much is known about what the unconscious<sup>1</sup> can or cannot do and about the processes by which the unconscious does its job. For instance, it is often taken as a fact that the unconscious cannot think rationally (DeWall, Baumeister, & Masicampo, 2008). Even the unconscious thought theory, which went far in explaining what the unconscious can perform, posits that the unconscious cannot do arithmetic or follow rules (Dijksterhuis & Nordgren, 2006). In contrast to this view, our contention is that the unconscious can initiate and follow simple arithmetic rules.

We need to state explicitly what we mean by rules in this context. We define rules as production rules that are “if then” pairs (Anderson, 1993). Applied to simple arithmetic, it can take the form of “if *instruction* and *Input 1*, *Input 2* then do *outcome*” (where *instruction* precedes *Inputs*) or alternatively in the arithmetic language “if *operator* and *Operand 1*, *Operand 2* then do *outcome*.” With simple arithmetic such as addition, the production rule could take the form of, for instance, “if *add* and 2, 3, then do 5.” We deliberately put the instruction (or operator) first and then inputs (operands) to capture the initiation of the production rule, instead of relying on a declarative knowledge. Doing so set our work apart from recent studies showing that participants exposed to masked equations such as “ $2 \times 3$ ” were faster at pronouncing a target number when it was the result of the equation (i.e., “6”) than when it was not (García-Orza, Damas-López, Matas, & Rodriguez, 2009). In this research, participants were indeed presented multiplicative equations in the very sequence ( $a \times b = c$ ) that they have been taught for a long time (i.e., learning multiplication tables). Thus, mere reliance on declarative knowledge can lead to the completion of a well-known sentence (e.g., “ $2 \times 3 = 6$ ”; Roussel, Fayol, & Barrouillet, 2002). Thus, although in line with our hypothesis, these results cannot be taken as evidence that the unconscious can follow arithmetic rules.

Although no work has shown that such rules can be followed unconsciously, we believe it can for two main reasons. First, a variety of goals can be activated unconsciously (e.g., “cooperate,” Bargh, Gollwitzer, Lee-Chai, Barndollar, & Trötschel, 2001; “remember,” Mitchell, Macrae, Schooler, Rowe, & Milne, 2002). Second, operators such as “add” have precisely been conceptual-

---

François Ric, Department of Psychology, University of Bordeaux, Bordeaux Cedex, France; Dominique Muller, Interuniversity Laboratory of Psychology, Pierre Mendes France University at Grenoble, Grenoble, France, and University Institute of France.

Correspondence concerning this article should be addressed to François Ric, Département de psychologie, Université de Bordeaux 2, 3ter, place de la Victoire 33076, Bordeaux Cedex, France, or Dominique Muller, Laboratoire Inter-universitaire de Psychologie, Université Pierre Mendès France, UFR SHS, 1251 av. Centrale BP 47 38 040, Grenoble Cedex 9, France. E-mail: francois.ric@u-bordeaux2.fr or dominique.muller@upmf-grenoble.fr

---

<sup>1</sup> We use the term *the unconscious* to simplify our message, but we conceptualize unconsciousness as a state of nonconsciousness, not as an independent entity.

ized as goals in the context of a production rule (Roussel et al., 2002; Sohn & Carlson, 1998). If it is, the social psychology literature suggests it could be activated unconsciously (see e.g., Fitzsimons & Bargh, 2004).

A stringent test of our hypothesis requires showing that the unconscious can apply simple arithmetic rules even when (a) the arithmetic operation does not require mere reliance on declarative knowledge; (b) the instruction, Input 1, and Input 2 are not presented in the form of an equation; (c) the task is not explicitly related to arithmetic; and (d) doing arithmetic is of no use in the task (Tzelgov, 1997). To meet these requirements, we developed an experimental design in which participants were given (or not) the instruction to add objects. Addition was chosen because it is less prone to declarative knowledge than is multiplication (see e.g., Roussel et al., 2002) and thus provides a more conservative test of our hypothesis. Participants received the instruction to add before the presentation of the inputs (i.e., not in the form of an equation). It is worth noting that such a situation is closer to everyday life computations, in which the goal to add is usually activated before knowing what has to be added. To make arithmetic irrelevant, the task was presented as a categorization task (i.e., a task to decide whether a symbol was either a digit or a letter) in which doing arithmetic was of no use to predict the category of the forthcoming symbol. In Experiment 1, the instruction was presented subliminally while inputs were presented supraliminally. In Experiments 2 and 3, both the instruction and inputs were presented subliminally.

## Experiment 1

### Method

**Participants.** Sixteen psychology students (13 women) participated in exchange for partial course credit.

**Procedure.** Participants were seated 1 m from a 17-in. 85-Hz computer screen. They were instructed that their task was to decide as accurately and fast as possible whether a target symbol appearing at the center of the screen was a number by pressing one of two keys (*A* or *P* on an AZERTY keyboard). The target was always preceded by a prime and two digits. The sequence began with a fixation point (800–1,200 ms) followed by a mask (MWMWM-WMWMWM) for 80 ms, the prime (*additionner* or *relativiser*; i.e., *add* or *relativize* in French) for 23 ms and another mask for 80 ms (see Figure 1, Panel A). Then, two digits were shown, one on each side of the fixation point (i.e., flankers), and remained there until the completion of the trial. After 1,200 ms, a target appeared in the center of the screen. The target was a number (i.e., 3, 4, 5, 6) for half of the trials and a letter (i.e., A, B, C, D) for the other half. When the target was a number, it was equal to the sum of the two digits in half of the trials, whereas it was not in the other half. Within each trial category, the flankers were preceded by one of the two primes in the same proportion (50%). After 10 practice trials, participants completed 112 experimental trials. It is critical to notice that in this situation (a) making additions was totally irrelevant because it was of no help to anticipate whether the target would be a number and (b) the configuration of the flankers was not predictive of the occurrence of a number (vs. a letter) because each flanker pair was equally associated with a number and with a letter. Yet if participants were summing the two digits, they should have the resulting number in mind as well as its properties (e.g.,

this is a number) when the target appeared. Thus, they should be faster to respond when the target is this very number compared with a different number. After they had completed the experiment, participants were carefully debriefed. Particular care was taken to assess whether participants reported having perceived any prime.

## Results and Discussion

None of the participants reported having even thought that a word had been presented at any time. However, a forced discrimination test was conducted on 18 other participants, who were presented the same material used in the main experiment (i.e., 112 trials) with the instruction to guess which of the two words (*add* or *relativize*) was presented in each trial (prime presentation was set up at 30 ms; see Experiments 2 and 3). Mean accuracy performance (51%) did not differ from chance level (50%;  $t < 1$ ), suggesting that participants were unable to identify the word *add* at such a brief presentation.

Analyses on accuracy revealed no significant effect ( $F_s < 2.5$ ,  $p_s > .14$ ). Response latencies on correct responses (errors = 4%; response latencies <300 ms or >1,000 ms [1.5%] were excluded) were inverse-transformed (Ratcliff, 1993) and submitted to a 2 (prime: *add* vs. *relativize*)  $\times$  2 (target: sum vs. not sum) repeated-measures analysis of variance (ANOVA). The predicted Prime  $\times$  Target interaction reached significance,  $F(1, 15) = 6.18$ ,  $p < .03$ ,  $\eta_p^2 = .29$ . Participants primed with *add* were faster at identifying a digit when the target was the sum of the flankers than when it was not,  $t(15) = 2.60$ ,  $p < .021$ ,  $\eta_p^2 = .31$  (see Figure 1, Panel C). No such difference appeared when participants were primed with *relativize*,  $t(15) = 1.25$ ,  $p < .23$ . We also found that when the target was the sum, participants were faster when primed with *add* than when they were primed with *relativize*,  $t(15) = 3.02$ ,  $p < .009$ ,  $\eta_p^2 = .38$ .

Consistent with our expectations, in trials where *add* was primed, participants were significantly faster at identifying the target as a number when it was equal to the sum of the two digits than when it was not. This effect was not found in trials where *relativize* was primed, suggesting that summing the digits was not the default option.

## Experiment 2

Experiment 1 reveals that simple arithmetic rules can be initiated unconsciously. In Experiment 2, we went one step further and tested whether such a rule can be not only initiated but also followed unconsciously. Because the to-be-added digits were presented supraliminally in Experiment 1, participants could have been partially aware that they were sometimes doing arithmetic, even if they did not know why. If the unconscious can initiate and follow simple rules, then people should be able to add digits without being aware they are doing so. To test this, we relied on the same procedure but this time with the to-be-added digits being presented subliminally, because previous research has shown that numerical stimuli can be primed without conscious awareness (see e.g., Bahrami et al., 2010). Replicating Experiment 1 results under these conditions would provide strong evidence that the unconscious can initiate and follow arithmetic rules. Moreover, we changed the control prime to avoid any interpretation relying on specific features of the verb used as control in Experiment 1.

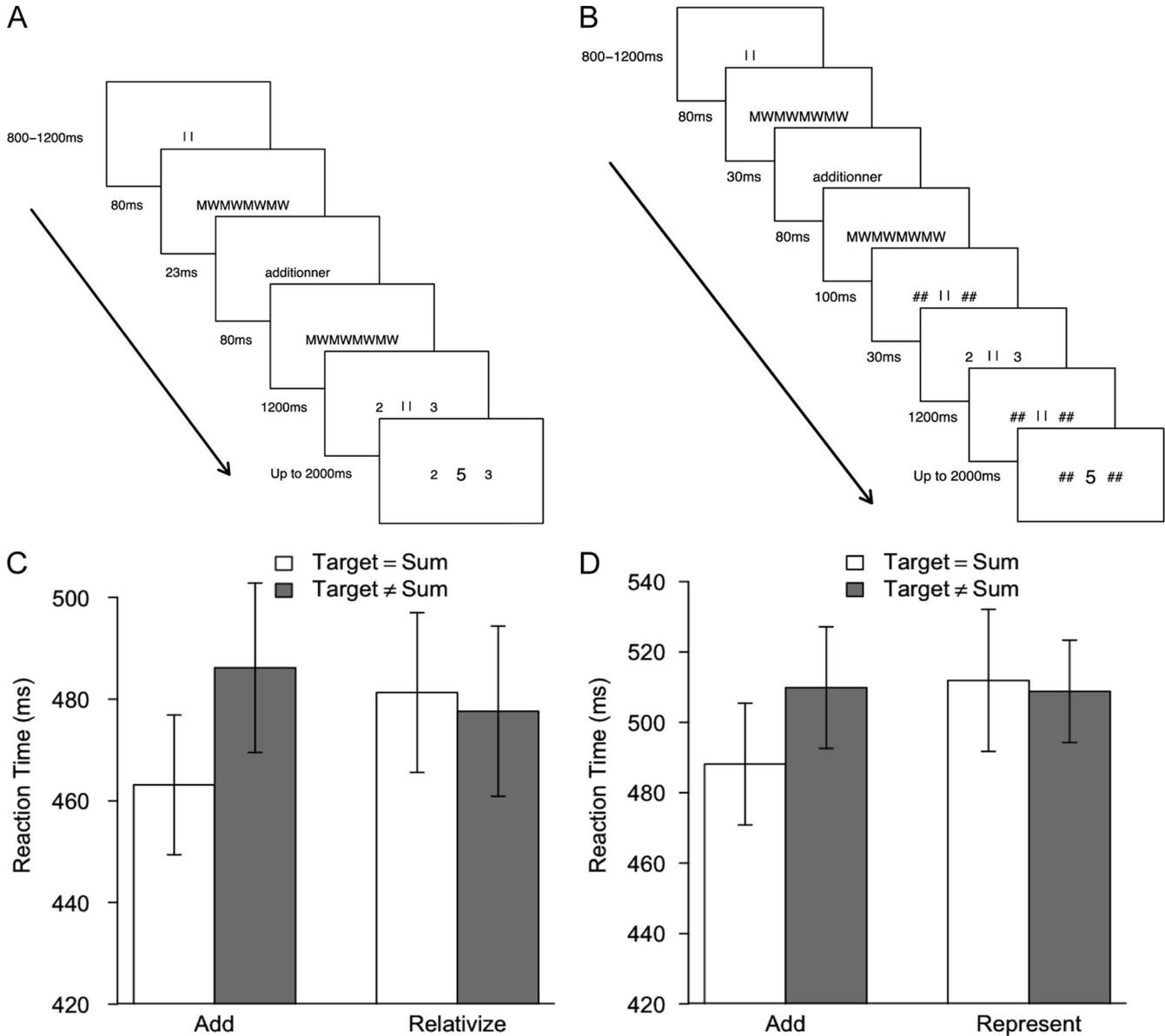


Figure 1. Sequence of events for each trial in Experiment 1 (Panel A) and Experiment 2 (Panel B) and untransformed mean reaction time for Experiment 1 (Panel C) and Experiment 2 (Panel D). Error bars represent the standard error.

## Method

**Participants.** Fifteen psychology students (14 women) participated in exchange for partial course credit.

**Procedure.** The procedure was similar to that in Experiment 1, except that participants were now primed with *add* versus *represent* (in French, *représenter*) and that both the primes and the flankers were presented for 30 ms (see Figure 1, Panel B). The flankers were positioned to be perceived at the same time (angular distance of 4.5°). These flankers were preceded (100 ms) and followed by a mask (##) that remained there until the completion of the trial. The target appeared 1,200 ms after the flankers. The

trials were terminated by the participants' response or automatically after 2,000 ms.

## Results and Discussion

None of the participants reported having seen words or digits. However, we conducted a forced discrimination test on the digits (for prime words, see Experiment 1). Sixteen other participants were presented the same procedure but with the instruction to guess which pair of digits (composed of two digits from 1 to 5) was presented on each of 112 trials. Mean accuracy ( $M = 7.47\%$ ) was slightly above chance level (4%),  $t(15) = 3.00$ ,  $p < .01$ .

Detection above chance level appeared on only three pairs, which were thus excluded from subsequent analyses (for all other pairs,  $t_s < 1.40$ ,  $p_s > .18$ ).

Analyses on accuracy revealed no significant effect (all  $F_s < 2.04$ ,  $p_s > .17$ ). Response latencies on correct responses were inverse-transformed (errors = 2.54%; reaction times <300 ms or >1,000 ms [1.78%] were excluded) and submitted to a 2 (prime: add vs. represent)  $\times$  2 (target: sum vs. not sum) repeated-measures ANOVA. The predicted Prime  $\times$  Target interaction reached significance,  $F(1, 14) = 5.30$ ,  $p < .04$ ,  $\eta_p^2 = .27$  (see Figure 1, Panel D). When primed with *add*, participants were faster when the target was the sum than when it was not,  $t(14) = 2.78$ ,  $p < .015$ ,  $\eta_p^2 = .36$ . No such difference appeared when participants were primed with *represent*,  $t(14) < 1$ . When the target was equal to the sum of the flankers, participants were faster when primed with *add* than when they were primed with *represent*,  $t(14) = 2.35$ ,  $p < .034$ ,  $\eta_p^2 = .28$ .

The results of this second experiment nicely replicate those observed in Experiment 1. This time, they were obtained with another control prime (*represent*) and with participants not being conscious of the digits they actually summed. These findings strengthen the contention that arithmetic rules can be initiated and followed unconsciously.

### Experiment 3

This study was conducted to replicate Experiment 2 with a more conservative control of the unconscious perception of the to-be-added digits. To do so, we presented the to-be-added digits for a shorter duration (20 ms instead of the 30 ms in Experiment 2), and awareness was checked with the use of a discrimination task completed by the participants who took part in the study. This procedure allows for evaluating whether the effects observed in Experiments 1 and 2 can be replicated while controlling for the idiosyncratic level of conscious perception for the to-be-added digits (see e.g., Greenwald, Klinger, & Schuh, 1995).

### Method

**Participants.** Thirty-two psychology students (all women) participated in exchange for partial course credit.

**Procedure.** The procedure was identical to Experiment 2, except for the following points. First, the flankers were presented for 20 ms instead of 30 ms (we used a 100-Hz computer screen). Second, immediately after the main task, participants completed the awareness forced-choice test (see Experiment 2) with the presentation of the flankers set at 20 ms.

### Results and Discussion

None of the participants reported having seen words or digits during the main task. Results on the forced-choice task revealed that participants were unable to detect the digits presented in the trials ( $M = 4.45\%$ ) above chance level (4%),  $t(31) = 1.29$ ,  $p > .20$ . Analyses on accuracy revealed no significant effect (all  $F_s < 2.06$ ,  $p_s > .16$ ).

Inverse-transformed latencies on correct answers (errors = 2.21%; reaction times <300 ms or >1,000 ms [1.83%] were excluded) were treated with a regression strategy<sup>2</sup> (Greenwald

et al., 1995). This strategy consists in testing each within-subject effect as the intercept of a regression having level of conscious perception (centered on the 0.04 value; i.e., random detection) as a predictor. By doing so, each effect is tested for a zero level of conscious perception (i.e., random detection). The Target  $\times$  Prime interaction reached significance,  $F(1, 30) = 4.85$ ,  $p < .04$ ,  $\eta_p^2 = .14$  (see Figure 2), and this interaction was not dependent on conscious perception (CP;  $F < 1$ ). When primed with *add*, participants were faster when the target was equal to the sum of the flankers than when it was not,  $t(30) = 4.05$ ,  $p < .001$ ,  $\eta_p^2 = .36$  (CP:  $t < 1$ ). This effect did not reach significance when participants were primed with *represent*,  $t(30) = 1.82$ ,  $ns$  (CP:  $t < 1$ ). Finally, when the target was equal to the sum of the flankers, participants were faster when primed with *add* than with *represent*,  $t(30) = 2.85$ ,  $p < .01$ ,  $\eta_p^2 = .21$  (CP:  $t < 1$ ). These results replicate the findings of Experiments 1 and 2 and further show that these effects stand when controlling for participants' conscious perception of the digits they are currently adding.

### General Discussion

Results of this research indicate that the unconscious can initiate and follow simple arithmetic rules. These effects were observed when the instructions were activated out of conscious awareness (Experiment 1) but also when it was the case for the instruction and the inputs to be added (Experiments 2 and 3). In line with a growing literature (Bargh, 2007), our results attest that the unconscious can do a lot more than we have thought for a long time (Dijksterhuis & Nordgren, 2006). Actually, the unconscious can do addition.

This research was intended to test whether the unconscious can follow simple arithmetic rules and therefore provides little insight on what could occur with more complex rules. It is worth noting, however, that one-digit addition is not so simple, because it requires the operation of executive functioning (see e.g., Deschuyteneer & Vandierendonck, 2005; Lemaire, Abdi, & Fayol, 1996) and relies more on procedural knowledge than other operations (e.g., one-digit multiplication; Roussel et al., 2002). Thus, we think our results are likely to apply to other simple arithmetic rules. We do not claim, however, that the unconscious can solve complex arithmetic problems such as  $13 \times 14$  (Dijksterhuis & Nordgren, 2006). Such problems indeed require the use of several steps and thus to articulate several rules (Ashcraft, 1992), a process that the unconscious may be unable to do. It is left to future research to explore which rules the unconscious can use and articulate. Such studies could have great implications for research addressing both unconscious processes and arithmetical reasoning.

These results also have implications for research on goal activation because they suggest that high-level mental processes implied in logical reasoning are not necessarily consciously launched and controlled. These experiments reveal that the goal of engaging in mental activity can be activated (primed) out of conscious awareness and pursued even when such activity is irrelevant to the task at hand. These results strongly support the possibility of an unconscious initiation of actions (Bargh, 1997). It is indeed diffi-

<sup>2</sup> Similar results were found using the same analytic procedure used in Experiments 1 and 2.

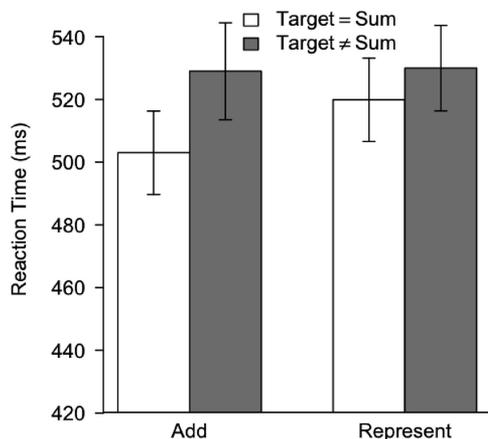


Figure 2. Adjusted means for untransformed reaction times in Experiment 3. Error bars represent the standard error.

cult to argue that this experimental situation requires a particular action whose use would have been strengthened by goal priming, as has often been the case in previous studies (Chartrand & Bargh, 1996; Holland, Hendriks, & Aarts, 2005). Such an interpretation is especially unlikely for Experiment 3 because we controlled for each participant's ability to perceive that there were digits to sum. Thus, these findings are consistent with previous theorizing on goal activation (Bargh, 1997; Dijksterhuis & Aarts, 2010). They extend this work by showing that goal pursuit can be initiated without contextual relevance; that it can imply high-level, rule-based, mental processes; and that it can lead people to use objects that they do not perceive consciously. In other words, high-level mental processes can operate totally unconsciously from activation to execution and finally completion.

## References

- Anderson, J. R. (1993). *Rules of the mind*. Hillsdale, NJ: Erlbaum.
- Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*, 75–106. doi:10.1016/0010-0277(92)90051-I
- Bahrami, B., Vetter, P., Spolaore, E., Pagano, S., Butterworth, B., & Rees, G. (2010). Unconscious numerical priming despite interocular suppression. *Psychological Science*, *21*, 224–233. doi:10.1177/0956797609360664
- Bargh, J. A. (1997). The automaticity of everyday life. In R. S. Wyer (Ed.), *Advances in social cognition* (Vol. 10, pp. 1–61). Mahwah, NJ: Erlbaum.
- Bargh, J. A. (Ed.). (2007). *Social psychology and the unconscious: The automaticity of higher mental processes*. New York, NY: Psychology Press.
- Bargh, J. A., Gollwitzer, P. M., Lee-Chai, A., Barndollar, K., & Trötschel, R. (2001). The automated will: Nonconscious activation and pursuit of behavioral goals. *Journal of Personality and Social Psychology*, *81*, 1014–1027. doi:10.1037/0022-3514.81.6.1014
- Chartrand, T. L., & Bargh, J. A. (1996). Automatic activation of impression formation and memorization goals: Nonconscious goal priming reproduces effects of explicit task instructions. *Journal of Personality and Social Psychology*, *71*, 464–478. doi:10.1037/0022-3514.71.3.464
- Deschuyteneer, M., & Vandierendonck, A. (2005). Are “input monitoring” and “response selection” involved in solving simple mental arithmetical sums? *European Journal of Cognitive Psychology*, *17*, 347–370. doi:10.1080/09541440440000032
- DeWall, C. N., Baumeister, R. F., & Masicampo, E. J. (2008). Evidence that logical reasoning depends on conscious processing. *Consciousness and Cognition*, *17*, 628–645. doi:10.1016/j.concog.2007.12.004
- Dijksterhuis, A. (2004). Think different: The merits of unconscious thought in preference development and decision making. *Journal of Personality and Social Psychology*, *87*, 586–598. doi:10.1037/0022-3514.87.5.586
- Dijksterhuis, A., & Aarts, H. (2010). Goals, attention, and (un)consciousness. *Annual Review of Psychology*, *61*, 467–490. doi:10.1146/annurev.psych.093008.100445
- Dijksterhuis, A., & Nordgren, L. F. (2006). A theory of unconscious thought. *Perspectives on Psychological Science*, *1*, 95–109. doi:10.1111/j.1745-6916.2006.00007.x
- Fitzsimons, G. M., & Bargh, J. A. (2004). Automatic self-regulation. In R. F. Baumeister & K. D. Vohs (Eds.), *Handbook of self-regulation: Research, theory, and applications* (pp. 151–170). New York, NY: Guilford Press.
- García-Orza, J., Damas-López, J., Matas, A., & Rodríguez, J. M. (2009). “2 × 3” primes “6”: Evidence from masked priming. *Attention, Perception, & Psychophysics*, *71*, 471–480. doi:10.3758/APP.71.3.471
- Greenwald, A. G., Klinger, M. R., & Schuh, E. S. (1995). Activation by marginally perceptible (“subliminal”) stimuli: Dissociation of unconscious from conscious cognition. *Journal of Experimental Psychology: General*, *124*, 22–42. doi:10.1037/0096-3445.124.1.22
- Holland, R. W., Hendriks, M., & Aarts, H. (2005). Smells like clean spirit: Nonconscious effects of scent on cognition and behavior. *Psychological Science*, *16*, 689–693. doi:10.1111/j.1467-9280.2005.01597.x
- Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, *8*, 73–104. doi:10.1080/095414496383211
- Mitchell, J. P., Macrae, C. N., Schooler, J. W., Rowe, A. C., & Milne, A. B. (2002). Unintentional remembering: Subliminal cues alter nonconscious memory strategies. *Memory*, *10*, 381–388. doi:10.1080/09658210244000207
- Ratcliff, R. (1993). Methods for dealing with reaction time outliers. *Psychological Bulletin*, *114*, 510–532. doi:10.1037/0033-2909.114.3.510
- Roussel, J. L., Fayol, M., & Barrouillet, P. (2002). Procedural vs. retrieval strategies in arithmetic: A comparison between additive and multiplicative problem solving. *European Journal of Cognitive Psychology*, *14*, 61–104. doi:10.1080/09541440042000115
- Sohn, M. H., & Carlson, R. A. (1998). Procedural framework for simple arithmetic skills. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *24*, 1052–1067. doi:10.1037/0278-7393.24.4.1052
- Tzelgov, J. (1997). Specifying the relations between automaticity and consciousness: A theoretical note. *Consciousness and Cognition*, *6*, 441–451. doi:10.1006/ccog.1997.0303

Received May 3, 2011

Revision received June 4, 2011

Accepted June 4, 2011 ■